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SUPERNEARNESS SPACES AS A TOOL FOR STUDYING UNIFICATION AND EXTENSIONS

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Topological extensions are closely related to nearness structures of various kinds. For example, the Smirnov compactification of a proximity space X is a compact Hausdorff space Y which contains X as a dense subspace and for which it is true that a pair of subsets of X is near if, and only if, their closures in Y meet. Lodato generalized this result to weaker conditions for the proximity and the space using *bunches* for the characterization of the extension.

Ivanova and Ivanov studied contiguity spaces and bicompact extensions such that a finite family of subsets are contigual if, and only if, there is a point of Y which is simultaneously in the closure in Y of each set of the family. Herrlich found a useful generalization of contiguity spaces by introducing nearness spaces, and Bentley showed that those nearness spaces which can be extended to topological ones have a neat internal characterization. Doitchinov introduced the notion of supertopological spaces in order to construct a unified theory of topological, proximity and uniform spaces, and he proved a certain relationship of some special classes of supertopologies—called b-supertopologies—with compactly determined extensions. Recently, supernear spaces were introduced by myself in order to define a common generalization of nearness spaces and supertopological spaces as well. As a basic concept, the notion of bounded sets is used, and functions (supernear operators) from a collection of bounded sets into powers of a given set are considered which fulfill certain axiom of nearness. So, we define for each bounded set the so-called *B-near collections*, and a bounded map is defined as a supernear-map if it preserves such collections. Thus, the new topological category **SN** whose objects are the supernear spaces is established, and moreover we can characterize those supernear spaces which can be extended to topological ones. As a basic notion we consider topological extensions which are strict in the sense of Banaschewski, where the hull in Y of the images of sets in X form a base for the closed subsets in Y. In this connection we study symmetrical and non-symmetrical extensions, and consequently we can characterize and describe those supernear spaces which are in a certain one-to-one relationship or correspondence, respectively, with them.

In other words, we describe those supernear spaces, whose B-near collections are determined by their respective constructing strict topological extension. As special cases we get back the former described constructions by various authors. So it seems to be possible to describe topological unification and extensions by the support of one *topological concept* with its corresponding maps.

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